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# Gauge Group Topology in Consistent Quantum Gravity


Mirjam Cvetič

Penn University of Pennsylvania

CAMTP University of Maribor

CERN

Univerza v Ljubljani Fakulteta za matematiko in fiziko

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Long history: [...Kumar, Taylor '09; Adams, DeWolfe, Taylor '10;... García-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura '17; Kim, Tarazi, Vafa '19; M.C., Dierigl, Lin, Zhang '20; Montero,Vafa '20; Hamada, Vafa '21; Tarazi, Vafa '21;...]

# Highlight

- Gauge symmetry topology for
  N =1 Supergravity in 8D →
  gauging of one-form symmetries

- Top-down classification via string junctions →
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# Guiding principles

- Geometry: primarily F-theory compactification
- Physics: global symmetries, including higher-form ones,
  gauged or broken in consistent quantum gravity
  [No Global Symmetry Hypothesis]

...[Harlow, Ooguri '18]

# Based on

- Gauge symmetry topology constraints in 8D

- M.C., M.Dierigl, L.Lin and H.Y.Zhang,
  ``String Universality and Non-Simply-Connected Gauge Groups in 8d,''
  PRL, arXiv:2008.10605 [hep-th];

- ``Higher-form Symmetries and Their Anomalies in M-/F-theory Duality,''
  PRD, arXiv:2106.07654 [hep-th] - 8D/7D & 6D/5D

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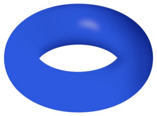
Digression: [Vafa'96;Morrison,Vafa'96],...review [Weigand'18]

# Key features of F-theory compactification

- F-theory, a powerful framework that geometrizes $\tau$ =axio-dilaton as a modular parameter of $T^2$ (SL(2,Z) duality of Type IIB string)



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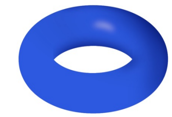
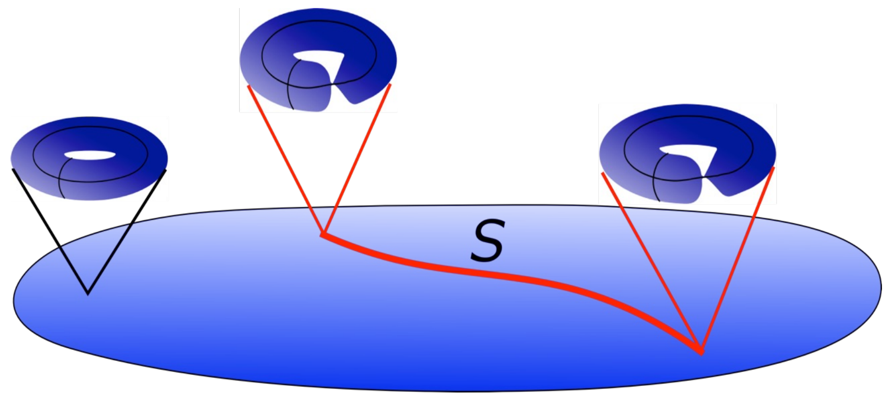


- Compactification on singular, elliptically fibered Calabi-Yau fewfolds



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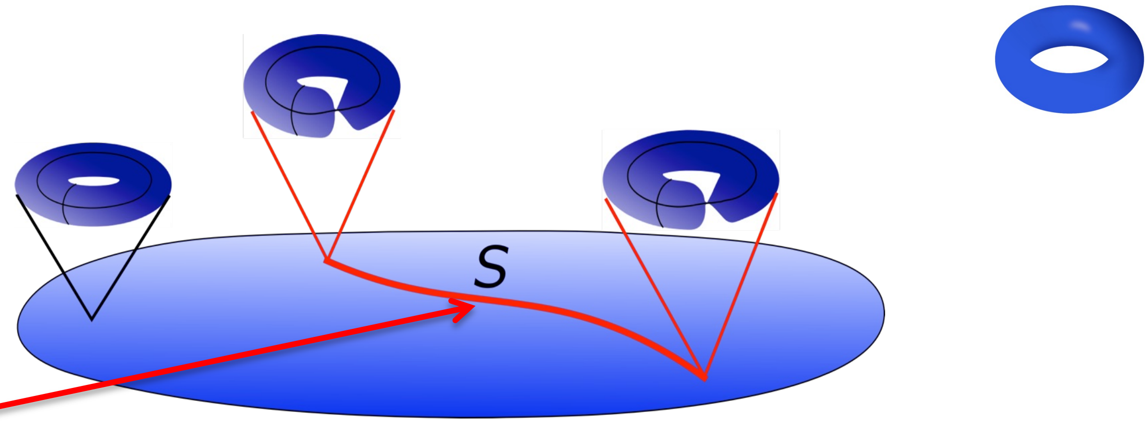


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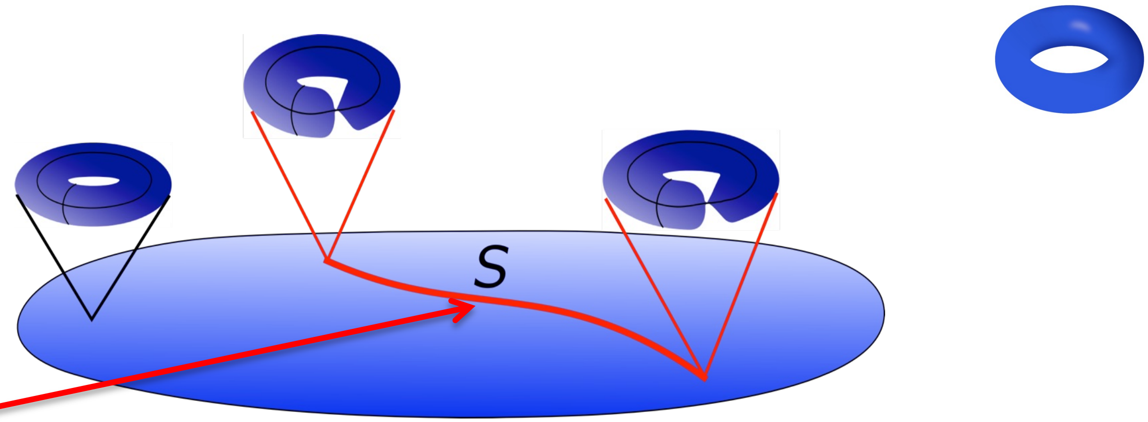


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- $T^2$ (elliptic curve) carries arithmetic structure: Mordell-Weil group of rational points → U(1)'s [Morrison,Park'12; M.C.,Klevers,Piragua'13; Borchmann, Mayrhofer,Palti,Weigand'13;...] torsional points → gauge group topology *Z* → *G/Z*
[Aspinwall,Morrison'98; Mayrhofer,Morrison,Till,Weigand'14; M.C.,Lin'17

F-theory compactification on elliptically fibered Calabi-Yau fourfolds led, for specific elliptic fibration to D=4 N=1effective theory with

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Current efforts: determination the exact matter spectra (including # of Higgs pairs) [Bies, M.C., Donagi,(Liu), Ong '21,'22]

[Bies, M.C., Donagi, Ong 2205.00008]

Matter spectra specified by root bundles ($K^{\{\text{frac no}\}}|_{\text{curve}}$) on matter curves:

Identified $O(10^{11})$ F-theory QSM geometries without vector-like matter exotics in the representations of $Q_L$ , $q_R$ , $e_R$

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- For $\Delta_4$ polytope ($10^{11}$ triangulations) 99.995% of root-bundles exactly $h^0 = 3$ → no vector-like exotics
- Statistical analysis for other polytopes → w/ $h^0 = 3$ by far most prevalent

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→ Study of Higgs nodal curves [Bies, M.C., Liu, work in progress]

No time, c.f. Martin Bies' talk

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# I. Gauge group topology in 8D N=1 SG

## a) Geometry - String compactification

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- *G* versus *G*/*Z* $\quad$ w/ $Z \subset Z(G)$-center
- For simplicity: $G = SU(n_1) \times SU(n_2) \times \ldots$
  w/ $Z(G) = \mathbb{Z}_{n1} \times \mathbb{Z}_{n2} \times \ldots$
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# Classification of allowed gauge groups in 8D N=1 SG

- Solutions to $\Sigma_i k^2_i (n_i - 1)/(2n_i) \in \mathbb{Z}$ , subject to $\Sigma_i (n_i - 1) = 18$

  [Montero, Vafa '20]

  limited. E.g., $G/\mathbb{Z}_\ell$ w/ $\ell > 8$ no anomaly-free solution;
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  [M.C., Dierigl, Lin, Zhang '21]

- Independently quantified by advancing string junction techniques including rank 2

  [M.C., Dierigl, Lin, Zhang '21 & '22]

# Classification of allowed gauge groups in 8D N=1 SG

- Solutions to $\Sigma_i k^2_i (n_i - 1)/(2n_i) \in \mathbb{Z}$ , subject to $\Sigma_i (n_i - 1) = 18$

  [Montero, Vafa '20]

  limited. E.g., $G/\mathbb{Z}_\ell$ w/ $\ell > 8$ no anomaly-free solution;
  unique solutions $\ell = 7$: $SU(7)^3/\mathbb{Z}_7$; $\ell = 8$: $[SU(8)^2 \times SU(4) \times SU(2)]/\mathbb{Z}_8$

- Also for other gauge groups & rank 10 and 2 theories.
  Confirmed in compactifications of CHL string (rank 10)

  [M.C., Dierigl, Lin, Zhang '21]

- Independently quantified by advancing string junction techniques
  including rank 2 [M.C., Dierigl, Lin, Zhang '21 & '22]

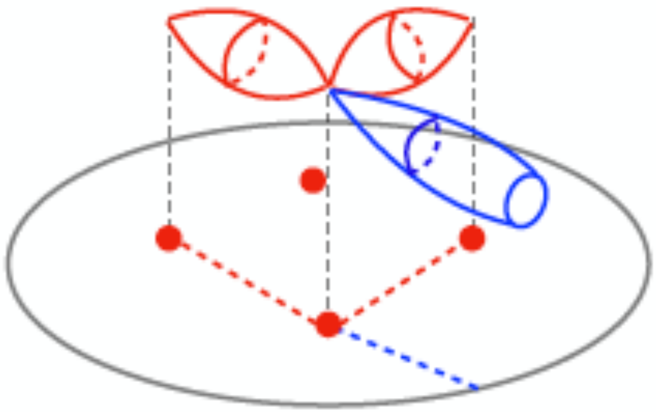
  → Long digression:

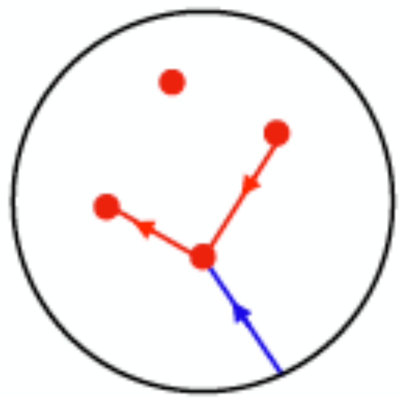
[M.C., Dierigl, Lin, Zhang 2203.03644]

# String Junctions & All Gauge Groups in 8D String Theory

String junctions between (p,q) 7-branes ⬌ geometry of 2-cycles

[Gaberdiel,Zwiebach '97, DeWolfe,Zwiebach '98]





in perpendicular
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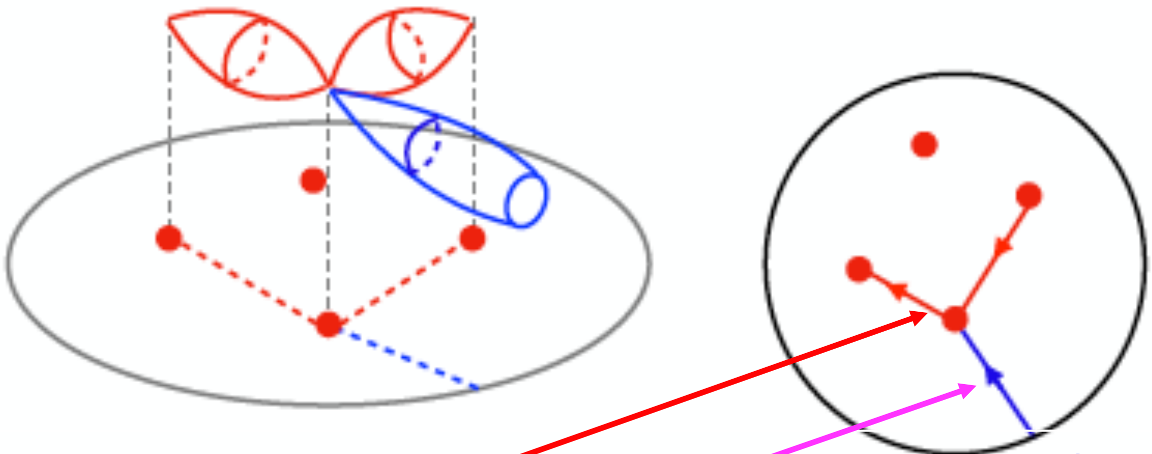

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String junctions w/ prongs on stack ⇔ roots of gauge algebra lattice

String junctions w/ external (asymptotic) prongs ⇔ weights

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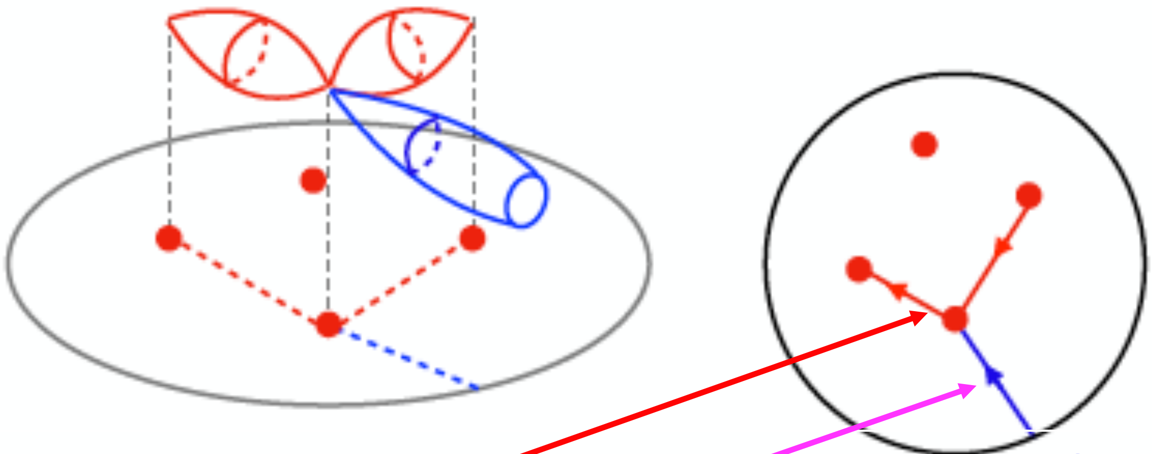

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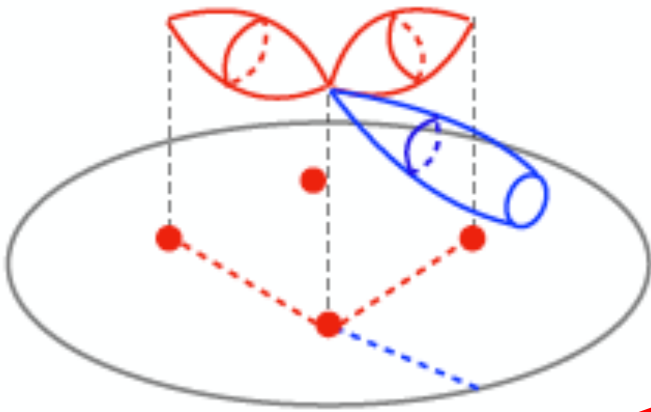

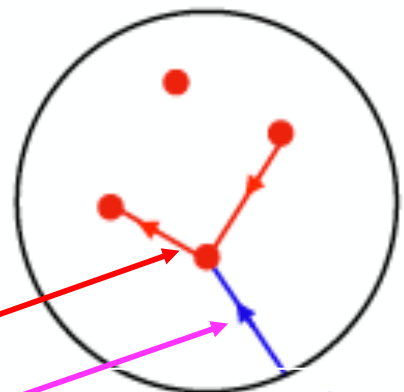

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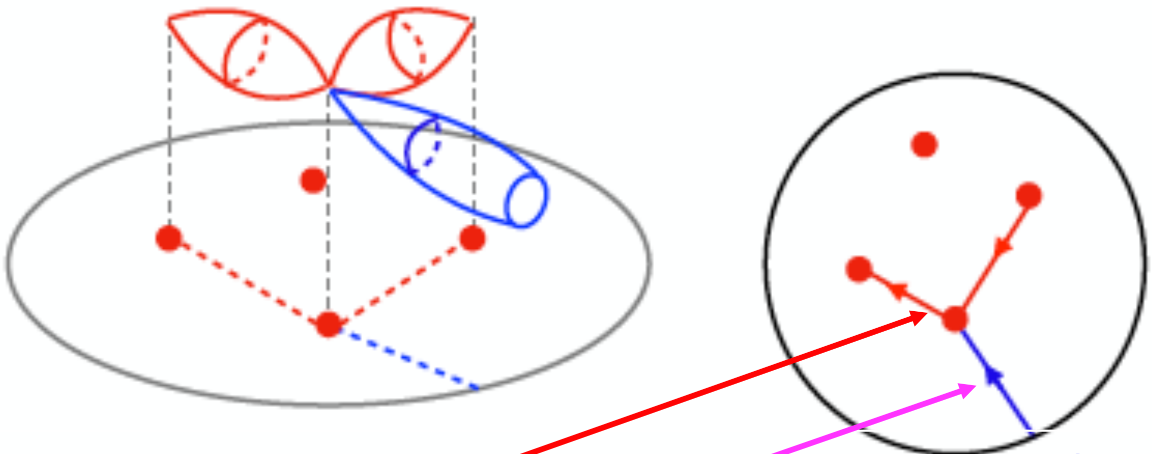

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$$\frac{\text{(co-)weights}}{\text{(co-)roots}} \Longleftrightarrow \frac{\text{non-compact}}{\text{compact 2-cycles}} = Z(G)\ !$$

(magnetic) electric higher-form symmetries

[Morrison,Schäfer-NamekiWillett ’20,
Albertini,Del Zotto,García-Etxebarria,Hosseini ’20]

## From local (non-compact) gauge group topology…

Non-root junctions carry non-zero asymptotic (p,q)-charge

$$\mathbf{j} = \lambda_i \boldsymbol{\alpha}_i + \boldsymbol{\omega}_{(p,q)} \ (\lambda_i \in \mathbb{Q})$$

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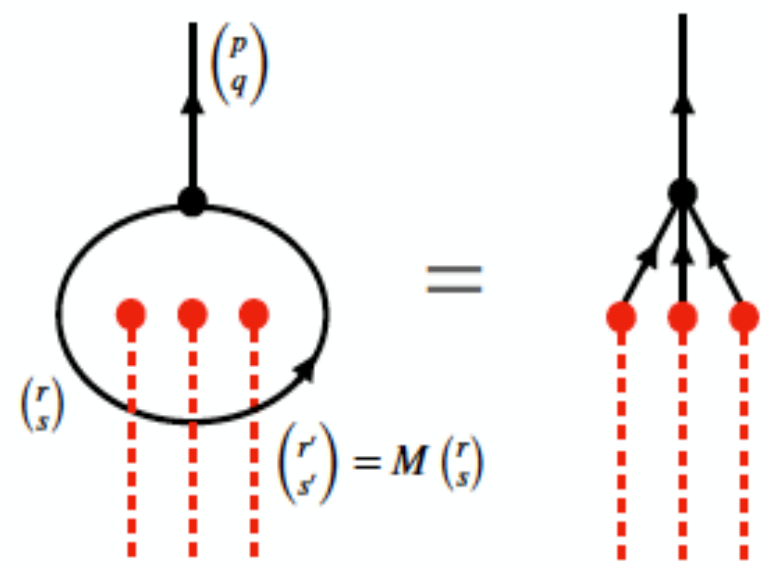
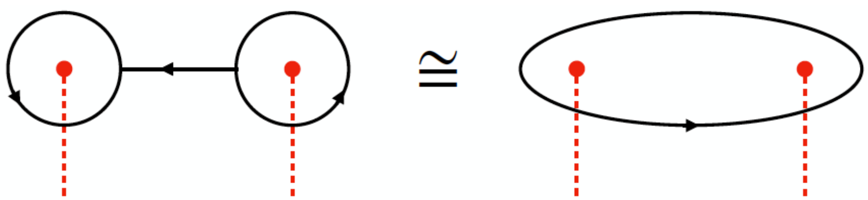
"Fractionality" of $\lambda_i \boldsymbol{\alpha}_i \equiv \mathbf{w}$ encodes charge under *Z(G)* → equivalently captured by *extended weights* $\boldsymbol{\omega}_{(p,q)}$ which *are fractional loop junctions*.



## …to global compactification & gauge group topology there

→ no net asymptotic (p,q) charge

→ restricts allowed junctions in "gluing" local patches encoded in fractional null junctions of 5-branes (encode *Z*)



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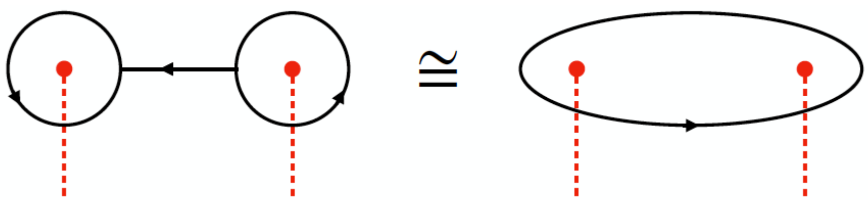

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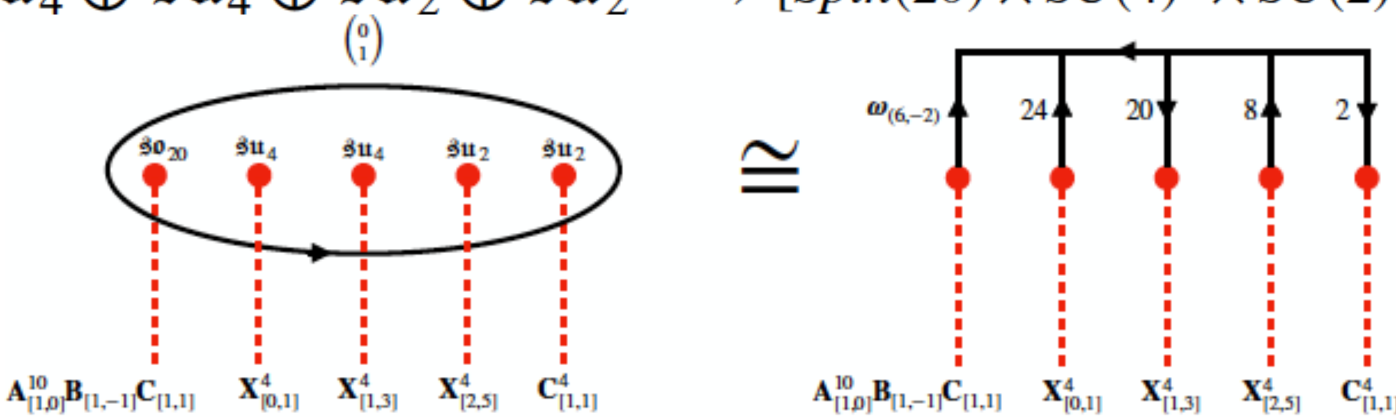
Example:

$$\mathfrak{g} = \mathfrak{so}_{20} \oplus \mathfrak{su}_4 \oplus \mathfrak{su}_4 \oplus \mathfrak{su}_2 \oplus \mathfrak{su}_2 \implies [Spin(20) \times SU(4)^2 \times SU(2)^2]/(\mathbb{Z}_2 \times \mathbb{Z}_2)$$



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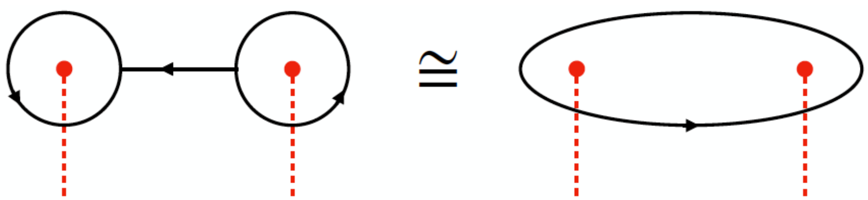

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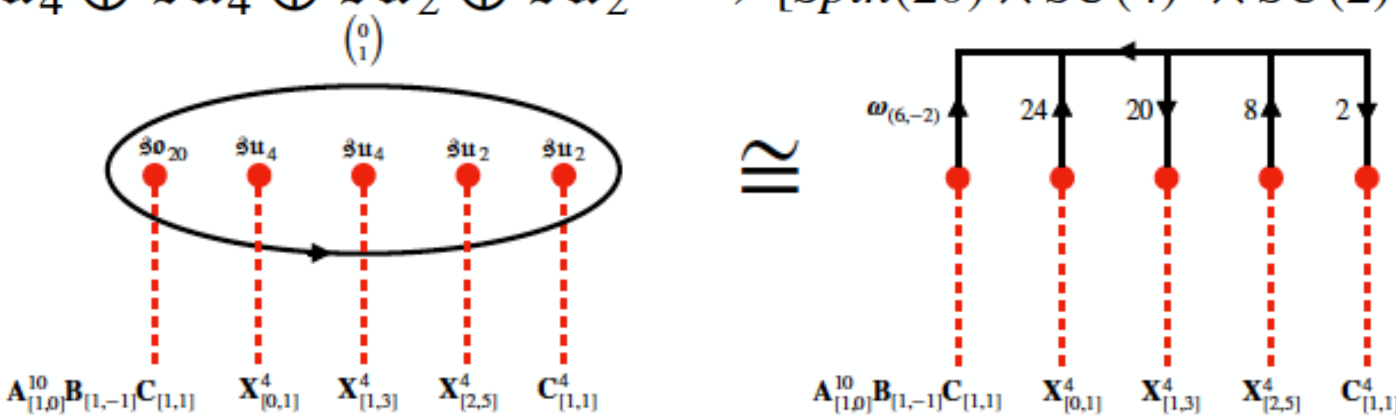


Also for all examples with U(1)'s

# Junctions on $O7^+$

- $O7^+$ - does not split into (p,q) 7-branes(unlike $O7^-$)

- Same monodromy as $\mathfrak{so}_{16}$ - stack, but w/ “non-trivial flux” that “freezes” singularity in M-/F-theory

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→ Analogous constructions w/global topology
w/ one $O7^+$ → all rank 10 vacua
w/ two $O7^+$ → all rank 2 vacua - first construction

# Junctions in 9D uplifts:
# sharpens swampland distance conjecture

- Suitable infinite distance limits of F-theory in *K3* moduli space describe 9D N=1 theories of rank 17

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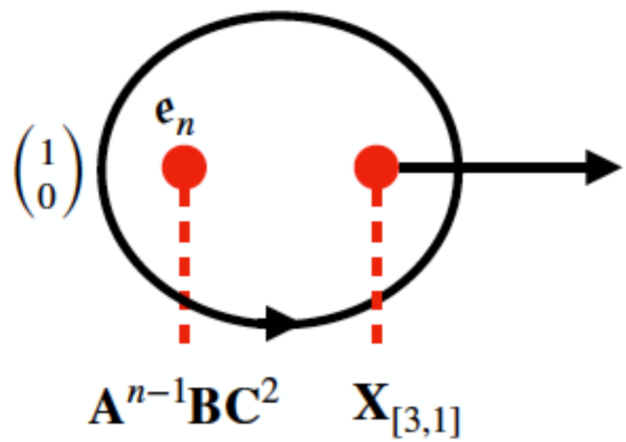

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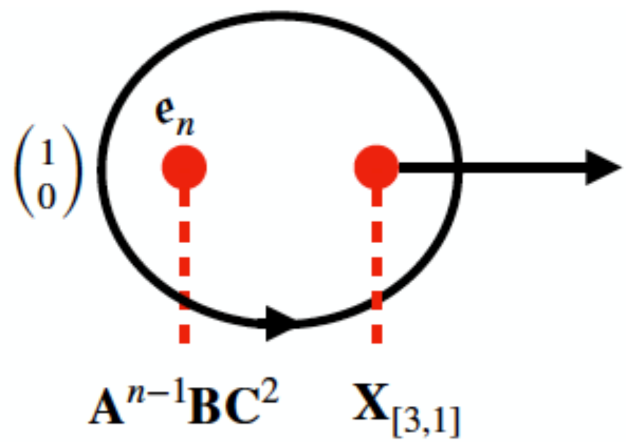


Two series:

$$\mathfrak{g}_{8d,\infty} = \mathfrak{su}_{18-m-n} \oplus \hat{\mathfrak{e}}_m \oplus \hat{\mathfrak{e}}_n \;\Rightarrow\; \mathfrak{g}_{9d} = \mathfrak{su}_{18-m-n} \oplus \mathfrak{e}_m \oplus \mathfrak{e}_n \,,$$

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[Maximal non-Abelian enhancement in D=9 heterotic vacua

[Font, Fraiman,Grana,Parra de Freitas '20]]

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- Characterized by ``freezing'' of one $\hat{e}_8$

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All 9D string vacua are "emergent" from 8D ones!

# Role of 1-form symmetry & Mixed 1-form - gauge anomalies in D≤8

- 8D [Font, Graña ,Fraiman, Freitas '21] – heterotic
  [M.C., Dierigl, Lin, Zhang '21, '22] – string junctions
- 7D [M.C., Dierigl, Lin, Zhang '21] – F/M-theory duality (torsional boundary $G_4$)
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- Mixed higher-form - gauge anomalies
  have important implications also for 6D and 5D SCFTs

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  Gauged 1-form symmetry in 8D

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- Their role in in quantum gravity - string theory on compact spaces

  [M. C., Heckman, Hübner, Torres to appear]

*Thank you!*